# REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188



| 1. AGENCY USE ONLY (Leave blank) | 2. REPORT DATE | 3. REPORT TYPE AND DATES COVERED |
|---|---|---|
| | 22 April 1996 | Final Technical Report, 950301-960229 |

**4. TITLE AND SUBTITLE**

Final Technical Report on AFOSR Grant F49620-95-1-0222,
COMPUTATION AND THEORY IN NONLINEAR OPTIMIZATION

**5. FUNDING NUMBERS**

G
F49620-95-1-0222

**6. AUTHOR(S)**

Stephen M. Robinson

**7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)**

University of Wisconsin-Madison
Madison, WI 53706

AFOSR-TR-96
0344

**9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)**

Air Force Office of Scientific Research
AFOSR/NM, Suite B115
110 Duncan Avenue
Bolling AFB, DC 20332-0001

**10. SPONSORING/MONITORING AGENCY REPORT NUMBER**

**11. SUPPLEMENTARY NOTES**

**12a. DISTRIBUTION/AVAILABILITY STATEMENT**

Approved for public release; distribution unlimited.

**12b. DISTRIBUTION CODE**

**13. ABSTRACT** *(Maximum 200 words)*

The principal objectives of this research were to investigate theoretical and computational properties of sample-path optimization (SPO) and collateral areas in nonsmooth optimization including the solution of variational inequalities. The progress achieved was documented in seven papers prepared for publication in journals or in edited volumes. In addition, two Ph.D. dissertations now in preparation are expected to acknowledge support from this AFOSR grant.

19960726 085

| 14. SUBJECT TERMS | 15. NUMBER OF PAGES |
|---|---|
| | 16. PRICE CODE |

| 17. SECURITY CLASSIFICATION OF REPORT | 18. SECURITY CLASSIFICATION | 19. SECURITY CLASSIFICATION OF ABSTRACT | 20. LIMITATION OF ABSTRACT |
|---|---|---|---|
| UNCLASSIFIED | UNCLASSIFIED | UNCLASSIFIED | UL |

**Final Technical Report**

*Computation and Theory in Nonlinear Optimization*

Grant No. F49620-95-1-0222

University of Wisconsin–Madison

## 1. Abstract

The principal objectives of this research project (as originally proposed for a longer effort) were to investigate the following questions related to the method of sample-path optimization (SPO) for stochastic systems:

1. Implementation of SPO on application problems and study of its computational behavior,
2. Development of local convergence conditions for SPO,
3. Investigating problem-specific properties for convergence of SPO,
4. Investigating rate of convergence of SPO,
5. Investigating appropriate deterministic optimization methods for use in connection with SPO,
6. Determining optimal tradeoffs between simulation run length and numbers of runs in SPO,
7. Performing collateral research in nonsmooth optimization in order to develop better methods for numerical optimization of functions that are not everywhere differentiable (including sample-path functions arising in SPO). This area includes methods for solving variational inequalities.

The progress achieved in these areas was documented in seven papers prepared for publication in journals or in edited volumes. In addition, two Ph.D. dissertations now in preparation are expected to acknowledge AFOSR support.

## 2. Overview of Research Accomplished

The research program as originally proposed was for a period of several years. In the one-year period of this grant, progress was achieved in several areas, including the following:

a. Convergence conditions for the method of sample-path optimization (SPO) were extended to cover local as well as global optimizers.

b. SPO was implemented for the problem of buffer-size optimization in tandem queues (e.g. manufacturing lines). This is a very hard problem, with discontinuous sample functions.

c. We began to apply SPO to economic decision problems, beginning with relatively simple cases such as option pricing. A great many important economic decision problems from industry and government can be regarded as instances of an abstract option-pricing problem. Progress on the solution of such problems should have many applications. Some of this research work is expected to be reported in the Ph.D. dissertation of A. Y. Özge.

d. We made considerable progress in collateral (supporting) studies involving nonsmooth optimization, which are needed to solve the SPO problems.

These areas correspond to Objectives 2, 1, 1, and 7 respectively. In the rest of this section we briefly describe this progress and provide citations to the publications listed in the next section. All are available either in the published literature, in press, or (in the case of papers in the publication process) from the principal investigator. (Note: References in brackets refer to numbered publications listed in Item 3.)

Convergence conditions for SPO were established in general form and the key elements needed for convergence were identified [P3]. The basic requirement is epiconvergence of the sample functions to the limit (steady state) function. It is shown in [P3] that this setup includes those used in previous numerical applications of this method in [P1] and in other papers.

Implementation of SPO for the problem of buffer-size optimization is reported in [P5]. Additional studies resulting from AFOSR-sponsored research will be reported in the Ph.D. dissertation of G. Gürkan.

A large numerical study of SPO was revised and the revised manuscript was accepted for publication [P1].

A method for global optimization of atomic configuration problems with Lennard-Jones potential functions was developed, and was reported in [P7] together with the results of numerical tests on problems with up to 20,000 atoms. This work was an outgrowth of previous AFOSR-sponsored research on optimization in chemistry.

A method for "collapsing" certain nonlinear variational inequalities was developed and justified [P2]. This method replaces the original problem by one in many fewer variables. In an example application a problem with 40 variables was reduced by this method to one with only 14 variables. The computing time to solve the reduced model was 2% of that required for the larger model.

A nonsmooth continuation method for solving nonlinear variational inequalities was successfully implemented [P4, P6]. The objective of this implementation was to produce a method that could successfully solve problems even if good starting points were not available. Tests were made on nonlinear optimization problems and on nonlinear variational

inequalities from the literature. The method appears to be robust and reliable, even with very bad starting points. The method has now been implemented as a solver for use with the GAMS modeling language, which should increase its usefulness.

### 3. Results from Research Activity

The following scientific works acknowledge support from Grant F49620-95-1-0222.

*Papers.*

[P1] E. L. Plambeck, B.-R. Fu, S. M. Robinson, and R. Suri, Sample-path optimization of convex stochastic performance functions, Revised version of March 1995, accepted by *Mathematical Programming*.

[P2] S. M. Robinson, A reduction method for variational inequalities, Version of August 1995. Submitted to *Mathematical Programming* and now undergoing minor revision for resubmission.

[P3] S. M. Robinson, Analysis of sample-path optimization, Revised version of October 1995, Accepted by *Mathematics of Operations Research*.

[P4] H. Sellami and S. M. Robinson, Homotopies based on nonsmooth equations for solving nonlinear variational inequalities, Version of November 1995, forthcoming in: G. Di Pillo and F. Giannessi, eds., *Nonlinear Optimization and Applications,* Plenum Publishing Corp., New York and London, 1996.

[P5] G. Gürkan and A. Y. Özge, Sample-path optimization of buffer allocations in a tandem queue – Part I: Theoretical issues, Version of December 1995, submitted to *Management Science* and now in revision for resubmission.

[P6] H. Sellami and S. M. Robinson, Implementation of a continuation method for normal maps, Version of February 1996, accepted by *Mathematical Programming*.

[P7] K. Ritter, S. M. Robinson, and S. Schäffler, Global minimization of Lennard-Jones functions on transputer networks, Version of March 1996, forthcoming in *Institute for Mathematics and Its Applications (IMA) Proceedings on Large-Scale Optimization*, 1996.

### 4. Participating Professionals

The following professional personnel received salary support from Grant F49620-95-1-0222.

- Stephen M. Robinson, Professor
- Gül Gürkan, Research Assistant
- A. Yonca Özge, Research Assistant

## 5. Degrees Awarded

- Gül Gürkan, Research Assistant. No degrees received this period. Currently working on Ph.D. dissertation; expected completion date August 1996. This dissertation is expected to acknowledge AFOSR support.
- A. Yonca Özge, Research Assistant. Degree of Master of Science (Computer Sciences) received December 1995. Passed doctoral preliminary examination 18 December 1995. Currently working on Ph.D. dissertation; expected completion date May 1997. This dissertation is expected to acknowledge AFOSR support.

## 6. Inventions and Patent Disclosures

During the work under this grant, there were no inventions that appeared to have any patent possibilities. Other (non-patentable) discoveries are contained in the papers reported above.

## 7. Other Information

Further information about any of the activities reported above, or other aspects of this research program, can be obtained from the principal investigator, Stephen M. Robinson, at the Department of Industrial Engineering, University of Wisconsin–Madison, 1513 University Avenue, Madison, WI 53706-1539, telephone (608) 263-6862, fax (608) 262-8454, email `smr@cs.wisc.edu`.